Hydropower Plants: Generating and Pumping Units
Solved Problems: Series 5

Reversible Pump-turbine power plant

1) What is the role of the draft tube in a Francis-type reversible pump-turbine?

   It is used to recover the kinetic energy of the flow by decreasing the axial velocity of the flow and transforming it into potential energy.

2) What are the advantages of having a variable speed unit?

   Variable speed machines have the ability, as their name suggest, to operate at different rotational frequencies. Thus, they can reach higher efficiencies when the discharge needs adaptation, therefore working at BEP for a broader operating range. Thus, they provide additional flexibility for primary and secondary frequency control.

3) In Figure 1, identify the components of the reversible pump-turbine listed in Table 1.

Table 1: Components to identify

1	Impeller
2	Guide vanes
3	Blade
4	Draft tube cone
5	Shaft
6	Stay vanes
7	Spiral case

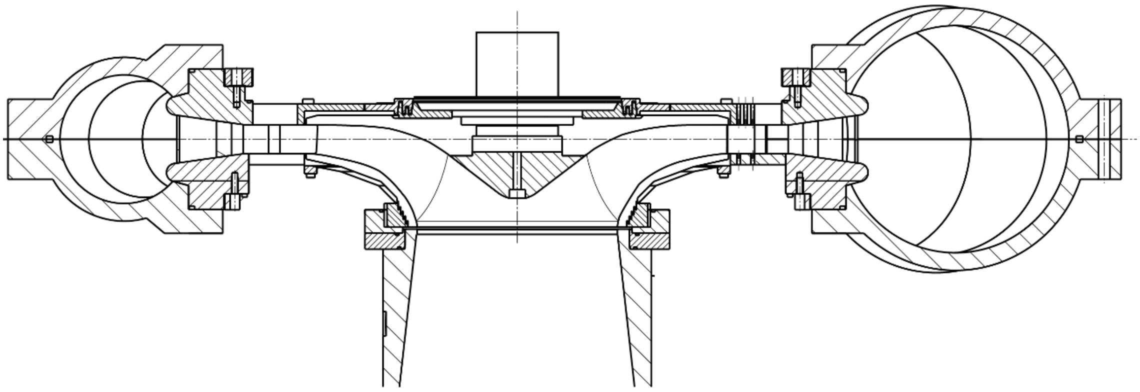


Figure 1: Cutview of the reversible pump-turbine

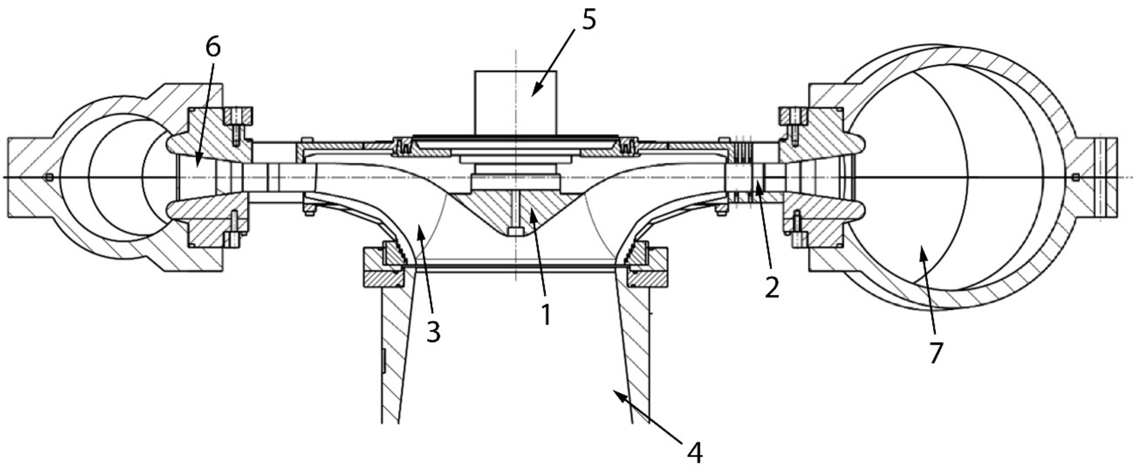


Figure 2: Answer to component identification

In the following questions, we are interested in the best efficiency point (BEP) of the reversible pump-turbine. The corresponding parameters of the power plant at BEP are given in Table 2.

Table 2: Parameters of the power plant at BEP

Variable	Unit	Value
Q	$(m^3 \cdot s^{-1})$	235
η_e	(-)	0.95
Z_B	(m)	750
$Z_{\bar{B}}$	(m)	573
N	(min^{-1})	128.6
D_{1e}	(m)	7.4
$D_{\bar{1}e}$	(m)	5.4
B	(m)	1
ρ	$(kg \cdot m^{-3})$	1000
g	$(m \cdot s^{-2})$	9.81

4) Compute the potential specific energy of the hydropower plant.

$$E_{potential} = g(Z_B - Z_{\bar{B}}) = 1736.4 \text{ J} \cdot \text{kg}^{-1}$$

5) For the rated discharge, the specific energy losses in the complete hydraulic circuit are estimated to $gH_r = 9.5 \text{ J} \cdot \text{kg}^{-1}$. Compute the available specific energy E.

$$E = E_{potential} - gH_r = g(Z_B - Z_{\bar{B}}) - gH_r = 1726.9 \text{ J} \cdot \text{kg}^{-1}$$

6) Compute the hydraulic power P_h.

$$P_h = \rho Q E = 405.82 \text{ MW}$$

7) Neglecting the mechanical losses and introducing the energy efficiency η_e, derive a relation between the final supplied power P and the hydraulic power P_h.

$$P = \eta_e P_h$$

8) Determine the final supplied power P and the torque T.

$$P = \eta_e P_h = 385.53 \text{ MW}$$

Knowing that $n = \dfrac{N}{60} = 2.14\ \mathrm{s^{-1}}$ *:*

$$T = \frac{P}{\omega} = \frac{P}{2\pi n} = 2.86 \cdot 10^{7}\ \mathrm{N \cdot m^{-1}}$$

9) Compute the external runner rotating velocity at the runner inlet and outlet, respectively U_{1e} and $U_{\bar{1}e}$.

$$U_{1e} = \omega R_{1e} = \omega \frac{D_{1e}}{2} = 49.84\ \mathrm{m \cdot s^{-1}}$$

$$U_{\bar{1}e} = \omega R_{\bar{1}e} = \omega \frac{D_{\bar{1}e}}{2} = 36.37\ \mathrm{m \cdot s^{-1}}$$

10) Compute the meridional component of the absolute flow velocity at the runner inlet and outlet, respectively Cm_{1e} and $Cm_{\bar{1}e}$.

Knowing that the flow is radial at the inlet of the pump-turbine, and axial at its outlet:

$$A_{1e} = \pi D_{1e} B = 23.25\ \mathrm{m^2}$$

$$\rightarrow Cm_{1e} = \frac{Q}{A_{1e}} = 10.11\ \mathrm{m \cdot s^{-1}}$$

$$A_{\bar{1}e} = \pi R_{\bar{1}e}^2 = \pi \frac{D_{\bar{1}e}^2}{4} = 22.90\ \mathrm{m^2}$$

$$\rightarrow Cm_{\bar{1}e} = \frac{Q}{A_{\bar{1}e}} = 10.26\ \mathrm{m \cdot s^{-1}}$$

11) Compute the transformed specific energy E_t.

$$E_t = \eta_e E = 1640.56\ \mathrm{J \cdot kg^{-1}}$$

12) Assuming solid body rotation, compute the peripheral component of the absolute flow velocity at the runner inlet and outlet, respectively Cu_{1e} and $Cu_{\bar{1}e}$.

Using the Euler equation for finding the relationship between the circumferential velocities at the inlet and outlet, and that $Cu_{\bar{1}e} = 0$ *as we are at BEP:*

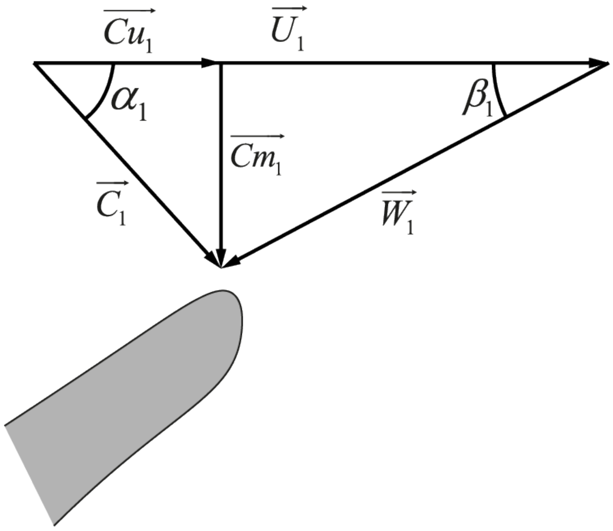
$$E_t = Cu_{1e} U_{1e} - \frac{1}{2} Cu_{\bar{1}e} U_{\bar{1}e} = Cu_{1e} U_{1e}$$

$$Cu_{1e} = \frac{E_t}{U_{1e}} = 32.92\ \ \mathrm{m \cdot s^{-1}}$$

13) Compute the absolute and relative flow angles at the runner inlet, α_{1e} and β_{1e} respectively, and sketch qualitatively the velocity triangle at the runner inlet.

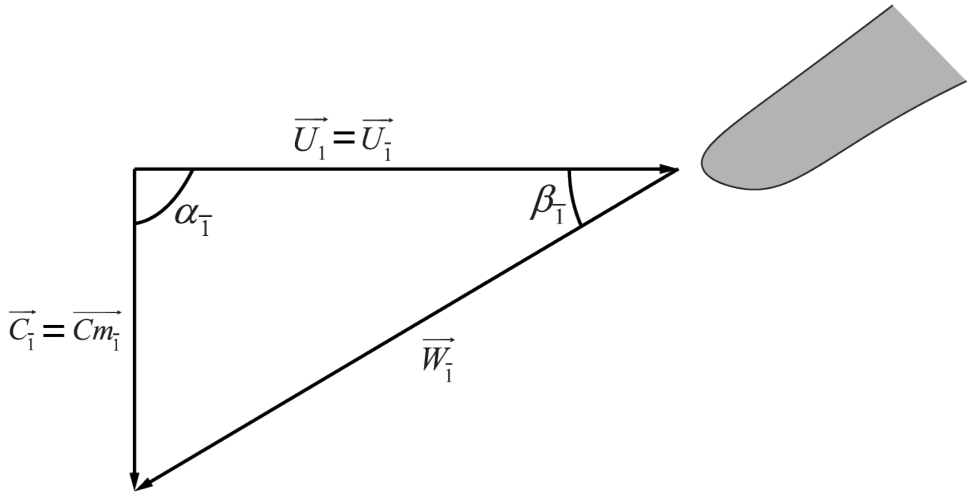
$$\tan \alpha_{1e} = \frac{Cm_{1e}}{Cu_{1e}} \quad \rightarrow \quad \alpha_{1e} = 17.07°$$

$$\tan \beta_{1e} = Cm_{1e} / (U_{1e} - Cu_{1e}) \quad \rightarrow \quad \beta_{1e} = 30.86°$$



Velocity triangle at the runner inlet

14) Now sketch qualitatively the velocity triangle at the runner outlet at the BEP.



Velocity triangle at the runner outlet - BEP

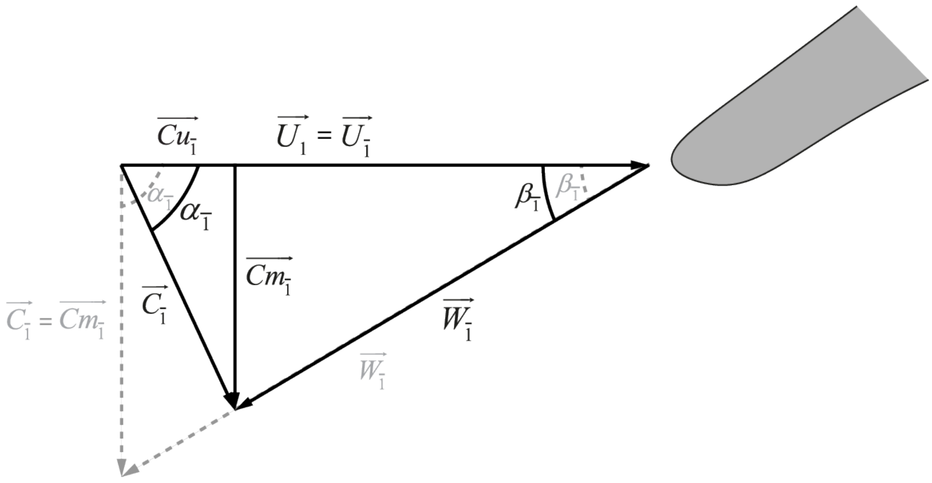
For the last part of this exercise, we want to study the pump-turbine at off-design conditions.

15) The operator wants to operate the machine at off-design conditions with a lower discharge to adjust the output power of the machine. This off-design condition is called part-load. How can they do that?

    They should decrease the opening angle of the guide vanes, as it's the only degree of freedom for fixed-speed machines.

16) The discharge value is now lower than the value at the Best Efficiency Point. Illustrate qualitatively the new situation with the velocity triangle at the runner outlet.

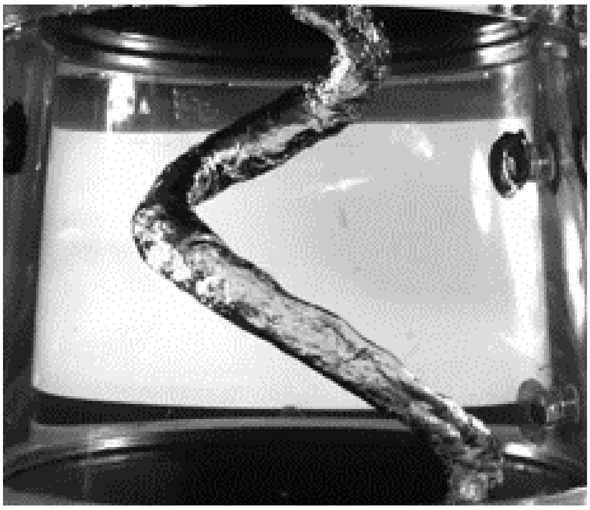
    At part-load, U_{1e} and $U_{\bar{1}e}$ remain equal and fixed, but $Cu_{\bar{1}e} \neq 0$ and the shape of the outlet velocity triangle changes accordingly. See the figure below, with the BEP components in dotted grey for comparison.



Velocity triangle at the runner outlet at part-load

17) What phenomenon can occur in the draft tube in this case?

A precessing vortex rope can develop. It can cause power surges, noise, erosion, efficiency loss, vibrations, etc.



Typical shape of a precessing vortex rope